

# 2026 MCM

2026 年 MCM (Mathematical Contest in Modeling, 数学建模竞赛)

## Problem A: Modeling Smartphone Battery Drain

题目 A: 智能手机电池耗电建模

Smartphones are indispensable tools in modern life, yet their battery behavior often seems unpredictable. On some days a phone may last the whole day; on other days it drains rapidly before lunch. Although some users attribute this to “heavy use,” the true drivers of battery depletion are more complex. Power consumption depends on the interplay of screen size and brightness, processor load, network activity, and background applications that continue drawing energy even when the device appears idle. Environmental conditions such as temperature further complicate matters: some batteries lose effective capacity in cold weather and may overheat under sustained heavy use. A battery’s behavior is also influenced by its history and how it has been charged during its lifetime.

智能手机已成为现代生活中不可或缺的工具，但其电池表现却常常显得难以预测。有时手机能续航一整天；有时却在午饭前就迅速掉电。尽管一些用户将其归因于“重度使用”，电量消耗的真实驱动因素要复杂得多。功耗取决于屏幕尺寸与亮度、处理器负载、网络活动以及后台应用等多种因素的相互作用——即便设备看似空闲，后台进程仍可能持续耗电。温度等环境条件会进一步增加复杂性：部分电池在寒冷环境下有效容量下降，而在持续高负载使用时可能出现过热。电池的表现还受到其使用历史以及整个生命周期充电方式的影响。

Your task is to develop a continuous-time mathematical model of a smartphone’s battery that returns the state of charge (SOC) as a function of time under realistic usage conditions. This will be used to predict the remaining time-to-empty under different conditions. You should assume that the phone has a lithium-ion battery.

你的任务是建立一个智能手机电池的连续时间数学模型，在真实使用条件下给出电池荷电状态 (SOC, state of charge) 随时间变化的函数关系。该模型将用于在不同条件下预测电池剩余可用时长（直至耗尽的时间）。请假设手机使用锂离子电池。

## Requirements:

要求:

1. Continuous-Time Model: Develop a model to represent the state of charge using a continuous-time equation or system of equations. You may want to begin with the simplest reasonable description of battery drain and then extend it to incorporate additional contributors such as screen usage, processor load, network connections, GPS usage, and other background tasks.

连续时间模型：使用连续时间方程或方程组建立模型来描述荷电状态 (SOC)。你可以从对电池耗电的最简合理描述出发，然后扩展以纳入屏幕使用、处理器负载、网络连接、GPS 使用以及其他后台任务等额外贡献因素。

Data as support, not substitute: You may collect or use data for parameter estimation and validation. If open datasets are limited, you may use published measurements or specifications (with proper citation), provided parameters are clearly justified and validated for plausibility. However, projects based solely on discrete curve fitting, timestep regression, or black-box machine learning without an explicit continuous-time model will not satisfy this problem’s requirements. All data used must be well documented and freely available, and the data must be free for use under an open license.

数据用于支撑，而非替代建模：你可以收集或使用数据进行参数估计与验证。若公开数据集有限，可使用已发表的测量结果或技术规格（须规范引用），但需对参数进行清晰论证，并验证其合理性。然而，仅基于离散曲线拟合、时间步回归，或缺乏明确连续时间模型的黑箱机器学习方法的项目，不符合本题要求。所有使用的数据必须有充分文档说明并可自由获取，且须在开放许可下允许自由使用。

2. Time-to-Empty predictions: Use your model to compute or approximate the time-to-empty under various initial charge levels and usage scenarios. Compare predictions to observed or plausible behavior, quantify uncertainty, and identify where the model performs well or poorly.

耗尽时间预测：使用你的模型在不同初始电量与不同使用情景下计算或近似得到“电量耗尽所需时间”。将预测结果与观测到的行为或合理的典型表现进行比较，量化不确定性，并指出模型在哪些方面表现良好、哪些方面表现欠佳。

- Show how your model explains differences in these outcomes and identify the specific drivers of rapid battery drain in each case.

展示你的模型如何解释上述不同结果之间的差异，并识别在每种情形下导致电量快速下降的具体驱动因素。

- Which activities or conditions produce the greatest reductions in battery life? Which ones change the model surprisingly little?

哪些活动或条件会使电池续航时间下降最多？哪些因素对模型输出的影响出乎意料地小？

3. Sensitivity and Assumptions: Examine how your predictions vary after making changes in your modeling assumptions, parameter values, and fluctuations in usage patterns.

敏感性与假设：研究当你的建模假设、参数取值以及使用模式的波动发生变化时，预测结果将如何随之改变。

4. Recommendations: Translate your findings into practical recommendations for a cellphone user. For example, which user behaviors—such as reducing brightness, disabling background tasks, or switching network modes—yield the largest improvements in battery life? How might an operating system implement more effective power-saving strategies based on insights from your model? Consider how battery aging reduces effective capacity or how your modeling framework could generalize to other portable devices.

建议：将你的发现转化为对手机用户具有可操作性的建议。例如，哪些用户行为——如降低亮度、禁用后台任务或切换网络模式——能带来最大的续航提升？操作系统可如何基于你模型的洞见实施更有效的省电策略？请考虑电池老化如何降低有效容量，以及你的建模框架如何推广到其他便携式设备。

## Your report should present:

报告应包含：

- A clear description of your model and governing equations.  
对你的模型及其控制方程的清晰描述。
- The assumptions and rationale behind your design choices.  
支撑你设计选择的假设与理由。
- Parameter estimation methods and validation results.  
参数估计方法与验证结果。
- A discussion of strengths, limitations, and possible extensions.  
对模型优势、局限性与可扩展方向的讨论。
- An executive-style summary highlighting main results, insights, and recommendations.  
一份“高层摘要”式的总结，突出主要结果、洞见与建议。

Important: Your model must be grounded in clearly defined physical or mechanical reasoning; discrete curve fitting or other mathematical forms that are disconnected from an explicit continuous-time description of battery behavior will not satisfy the requirements. Projects that rely solely on discrete curve fitting or

statistical regression without a clearly formulated continuous-time model will not satisfy the requirements of this problem.

重要提示：你的模型必须建立在清晰界定的物理或机理性推理基础之上；若仅进行离散曲线拟合，或采用与电池行为明确连续时间描述脱节的其他数学形式，将不满足要求。仅依赖离散曲线拟合或统计回归、且未清晰提出连续时间模型的项目，同样不符合本题要求。

Your PDF solution of no more than 25 total pages should include:

你的 PDF 解决方案（总页数不超过 25 页）应包含：

- One-page Summary Sheet.  
1 页摘要页 (Summary Sheet)。
- Table of Contents.  
目录。
- Your complete solution.  
完整解答。
- In-text Citations and A Reference List.  
文内引用与参考文献列表。
- AI Use Report (If used does not count toward the 25-page limit.)  
AI 使用报告 (如使用；不计入 25 页总页数限制)。

Note: There is no specific required minimum page length for a complete MCM submission. You may use up to 25 total pages for all your solution work and any additional information you want to include (for example: drawings, diagrams, calculations, tables). Partial solutions are accepted. We permit the careful use of AI such as ChatGPT, although it is not necessary to create a solution to this problem. If you choose to utilize a generative AI, you must follow the COMAP AI use policy. This will result in an additional AI use report that you must add to the end of your PDF solution file and does not count toward the 25 total page limit for your solution.

注：完整的 MCM 提交稿没有规定最低页数要求。你的全部解题内容及任何补充信息（例如：图示、示意图、计算、表格）最多可使用 25 页。允许提交部分解答。竞赛允许谨慎使用 ChatGPT 等 AI 工具，但完成本题并不依赖 AI。如你选择使用生成式 AI，必须遵循 COMAP 的 AI 使用政策；这将要求你在 PDF 解答文件末尾附加一份 AI 使用报告，该报告不计入 25 页总页数限制。

## Glossary

术语表

Smartphone: is a mobile device that combines the functionality of a traditional cell phone with advanced computing capabilities.

智能手机：一种将传统手机通信功能与高级计算能力相结合的移动设备。

Power Consumption: the rate at which a device uses electrical energy from its battery or power source.

功耗：设备从电池或电源中消耗电能的速率。

Processor Load: the actual amount of work being done by the processor at a given moment.

处理器负载：处理器在某一时刻实际执行的工作量水平。

State of Charge (SOC): a measure of how much energy remains in a battery compared to its full capacity, expressed as a percentage.

荷电状态 (SOC)：电池剩余能量相对于其满容量的比例度量，通常以百分比表示。

Time-to-Empty: the estimated amount of time remaining before a battery is completely discharged.  
耗尽时间 (Time-to-Empty): 电池完全放电之前预计剩余的时间长度。