2026 MCM Problem C: Data With The Stars

2026 MCM 题目 C：与星共舞的数据

Dancing with the Stars (DWTS) is the American version of an international television franchise based on the British show "Strictly Come Dancing"("Come Dancing"originally). Versions of the show have appeared in Albania, Argentina, Australia, China, France, India, and many other countries. The U.S. version, the focus of this problem, has completed 34 seasons.

《与星共舞》(Dancing with the Stars，DWTS）是一个国际电视节目品牌的美国版，源自英国节目"Strictly Come Dancing"(其前身为"Come Dancing")。该节目已在阿尔巴尼亚、阿根廷、澳大利亚、中国、法国、印度等多个国家推出不同版本。本题聚焦的美国版已播出 34 季。

Celebrities are partnered with professional dancers and then perform dances each week. A panel of expert judges scores each couple's dance, and fans vote (by phone or online) for their favorite couple that week. Fans can vote once or multiple times up to a limit announced each week. Further, fans vote for the star they wish to keep, but cannot vote to eliminate a star. The judge and fan votes are combined in order to determine which couple to eliminate (the lowest combined score) that week. Three (in some seasons more) couples reach the finals and in the week of the finals the combined scores from fans and judges are used to rank them from 1^{st} to 3^{rd} (or 4^{th} , 5^{th}).

明星与专业舞者配对，每周进行舞蹈表演。由专家评委团为每对选手评分，同时观众通过电话或网络为当周最喜欢的组合投票。观众每周可以投一次或多次票，但有当周公布的投票上限。此外，观众只能投票支持希望保留的明星，不能投票"淘汰"某位明星。评委评分与观众投票会被合并以确定当周被淘汰的组合（合并得分最低者)。最终有三对（部分季数更多）进入决赛，在决赛周，评委与观众的合并得分用于将他们从 1^{st} 排名到 3^{rd} （或 4^{th} 、5^{th})。

There are many possible methods of combining fan votes and judge scores. In the first two seasons of the U.S. show, the combination was based on ranks. Season 2 concerns (due to celebrity contestant Jerry Rice who was a finalist despite very low judge scores) led to a modification to use percentages instead of ranks. Examples of these two approaches are provided in the Appendix.

合并观众投票与评委评分有多种可能方式。在美国版前两季，合并方式以"排名"为基础。第二季因名人选手 Jerry Rice 尽管评委评分很低仍进入决赛而引发争议，促使规则改为使用"百分比"而非"排名"。附录提供了这两种方法的示例。

In season 27, another "controversy"occurred when celebrity contestant Bobby Bones won despite consistently low judges scores. In response, starting in season 28 a slight modification to the elimination process was made. The bottom two contestants were identified using the combined judge scores and fan votes, and then during the live show the judges voted to select which of these two to eliminate. Around this same season, the producers also returned to using the method of ranks to combine judges scores with fan votes as in seasons one and two. The exact season this change occurred is not known, but it is reasonable to assume it was season 28.

第 27 季出现了另一场"争议"：名人选手 Bobby Bones 尽管评委评分持续偏低却最终夺冠。为此，从第 28 季开始，淘汰流程进行了小幅调整：先依据评委评分与观众投票的合并结果确定得分最低的两对选手，然后在直播中由评委投票决定淘汰其中一对。大约在同一时期，制作方也回到了第 1、2 季使用的"排名法"来合并评委评分与观众投票。具体变更发生于哪一季并不确定，但合理的假设是第 28 季。

Judge scores are meant to reflect which dancers are technically better, although there is some subjectivity in what makes a dance better. Fan votes are likely much more subjective, influenced by the quality of the dance, but also the popularity and charisma of the celebrity. Show producers might actually prefer, to some extent, conflicts in opinions and votes as such occurrences boost fan interest and excitement.

评委评分旨在反映舞者的技术水平高低，但"何为更好"的判断仍带有一定主观性。观众投票可能更具主观性，既受舞蹈质量影响，也受明星人气与个人魅力左右。节目制作方在一定程度上可能并不排斥评委与观众之间的分歧，因为此类冲突会提升粉丝关注度与节目热度。

Data with judges scores and contestant information is provided and described below. You may choose to include additional information or other data at your discretion, but you must completely document the sources. Use the data to:

题目提供并在下文说明了评委评分与选手信息数据。你可自行选择加入其他信息或数据，但必须完整注明来源。使用这些数据来：

- Develop a mathematical model (or models) to produce estimated fan votes (which are unknown and a closely guarded secret) for each contestant for the weeks they competed.

  建立数学模型（或多个模型），为每位参赛者在其参赛的各周估计观众投票数（真实票数未知且高度保密）。

- Does your model correctly estimate fan votes that lead to results consistent with who was eliminated each week? Provide measures of the consistency.

- How much certainty is there in the fan vote totals you produced, and is that certainty always the same for each contestant/week? Provide measures of your certainty for the estimates.

  你的模型是否能估计出与每周淘汰结果一致的观众投票数？请提供一致性度量。
  你对所生成的观众投票总数有多大置信度？这种置信度在不同选手/不同周是否一致？请给出置信度度量。

- Use your fan vote estimates with the rest of the data to:

  使用你估计的观众投票并结合其余数据来：

Compare and contrast the results produced by the two approaches used by the show to combine judge and fan votes (i.e. rank and percentage) across seasons (i.e. apply both approaches to each season). If differences in outcomes exist, does one method seem to favor fan votes more than the other?

Examine the two voting methods applied to specific celebrities where there was "controversy", meaning differences between judges and fans. Would the choice of method to combine judge scores and fan votes have led to the same result for each of these contestants? How would including the additional approach of having judges choose which of the bottom two couples to eliminate each week impact the results? Some examples you might consider (there may also be others you identified):

比较并对照节目使用的两种合并评委与观众投票的方法（即排名法与百分比法）在各季产生的结果（即对每一季都应用两种方法）。若结果存在差异，是否有一种方法相对更偏向观众投票？

针对存在"争议"的特定名人（即评委与观众意见存在明显分歧者）比较两种投票方法。不同的合并方法会否导致这些选手的结果相同？若再加入"评委从最后两名中选择淘汰对象"的额外机制，又会如何影响结果？你可参考以下案例（也可加入你识别的其他案例）：

- season 2 - Jerry Rice, runner up despite the lowest judges scores in 5 weeks.

- season 4 - Billy Ray Cyrus was 5^{th} despite last place judge scores in 6 weeks.

- season 11 - Bristol Palin was 3^{rd} with the lowest judge scores 12 times.

- season 27 - Bobby Bones won the despite consistently low judges scores

  第 2 季——Jerry Rice：尽管有 5 周评委最低分，仍获得亚军。
  第 4 季——Billy Ray Cyrus：尽管有 6 周评委排名末位，仍获得 5^{th} 。
  第 11 季——Bristol Palin：评委最低分出现 12 次，最终排名 3^{rd} 。
  第 27 季——Bobby Bones：尽管评委评分持续偏低仍夺冠。

- Based on your analysis, which of the two methods would you recommend using for future seasons and why? Would you suggest including the additional approach of judges choosing from the bottom two couples?

  基于你的分析，你会推荐未来赛季使用哪一种方法，为什么？你是否建议加入"评委从最后两名中选择淘汰对象"的额外机制？

- Use the data including your fan vote estimates to develop a model that analyzes the impact of various pro dancers as well as characteristics for the celebrities available in the data (age, industry, etc). How

much do such things impact how well a celebrity will do in the competition? Do they impact judges scores and fan votes in the same way?

- Propose another system using fan votes and judge scores each week that you believe is more "fair"(or "better"in some other way such as making the show more exciting for the fans). Provide support for why your approach should be adopted by the show producers.

- Produce a report of no more than 25 pages with your findings and include a one- to two-page memo summarizing your results with advice for producers of DWTS on the impact of how judge and fan votes are combined with recommendations for how to do so in future seasons.

  使用包括你所估计观众投票在内的数据，建立模型分析专业舞者与名人特征（年龄、行业等）对比赛成绩的影响。这些因素对选手表现的影响程度有多大？它们对评委评分与观众投票的影响是否一致？
  提出一个你认为更"公平"(或在其他方面更"好"，例如让节目更精彩）的、基于每周评委评分与观众投票的替代方案，并给出理由说明为何该方案应被制作方采纳。
  形成不超过 25 页的报告，给出你的发现，并附上一至两页的备忘录，总结结果并向 DWTS 制作方提出关于评委与观众投票合并方式的建议以及未来赛季的推荐做法。

Your PDF solution of no more than 25 total pages should include:

你的 PDF 解答（总页数不超过 25 页）应包括：

One-page Summary Sheet.
Table of Contents.
- Your complete solution.
One- to two-page memo.
- References list.
AI Use Report (If used does not count toward the 25-page limit.)

1 页摘要页（Summary Sheet)。
目录。
- 完整解答。
一至两页的备忘录。
- 参考文献列表。
AI 使用报告（如使用；不计入 25 页总页数限制。）

Note: There is no specific required minimum page length for a complete MCM submission. You may use up to 25 total pages for all your solution work and any additional information you want to include (for example: drawings, diagrams, calculations, tables). Partial solutions are accepted. We permit the careful use of AI such as ChatGPT, although it is not necessary to create a solution to this problem. If you choose to utilize a generative AI, you must follow the COMAP AI use policy. This will result in an additional AI use report that you must add to the end of your PDF solution file and does not count toward the 25 total page limit for your solution.

注：完整的 MCM 提交稿没有规定最低页数要求。你的全部解题内容及任何补充信息（例如：图示、示意图、计算、表格）最多可使用 25 页。允许提交部分解答。竞赛允许谨慎使用 ChatGPT 等 AI 工具，但完成本题并不依赖 AI。如你选择使用生成式 AI，必须遵循 COMAP 的 AI 使用政策；这将要求你在 PDF 解答文件末尾附加一份 AI 使用报告，该报告不计入 25 页总页数限制。

Data File: 2026_MCM_Problem_C_Data.csv –contestant information, results, and judges scores by week for seasons 1 –34. The data description is provided in Table 1.

数据文件：2026_MCM_Problem_C_Data.csv ——包含第 1 至 34 季的参赛者信息、赛季结果及按周的评委评分。数据说明见表 1。

Table 1: Data Description for 2026_MCM_Problem_C_Data.csv

表 1：2026_MCM_Problem_C_Data.csv 数据说明

Variables	Explanation	Example
celebrity_name	Name of celebrity contestant (Star)	Jerry Rice, Mark Cuban, ⋯
ballroompartner	Name of professional dancer partner	Cheryl Burke, Derek Hough, ⋯
celebrity_industry	Star profession category	Athlete, Model, ⋯
celebrity_homestate	Star home state (if from U.S.)	Ohio, Maine, ⋯
celebrity_homecountry/region	Star home country/region	United States, England, ⋯
celebrity_age during season	Age of the star in the season	32, 29, ⋯
season	Season of the show	1, 2, 3, ⋯, 32
results	Season results for the start	1st Place, Eliminated Week 2, ⋯
placement	Final place for the season (1 best)	1, 2, 3, ⋯
weekXjudgeY_score	Score from judge Y in week X	1, 2, 3, ⋯

变量	说明	示例
celebrity_name	名人参赛者（明星）姓名	Jerry Rice, Mark Cuban, ⋯
ballroompartner	专业舞者搭档姓名	Cheryl Burke, Derek Hough, ⋯
celebrity_industry	明星职业类别	Athlete, Model, ⋯
celebrity_homestate	明星所在州（若来自美国）	Ohio, Maine, ⋯
celebrity_homecountry/region	明星所属国家/地区	United States, England, ⋯
celebrity_age during season	该季明星年龄	32, 29, ⋯
season	节目季数	1, 2, 3, ⋯, 32
results	该季结果	1st Place, Eliminated Week 2, ⋯
placement	该季最终名次（1 为最佳）	1, 2, 3, ⋯
weekXjudgeY_score	第 X 周评委 Y 的评分	1, 2, 3, ⋯

Notes on the data:

数据说明:

1. Judges scores for each dance are from 1 (low) to 10 (high).

   每支舞的评委评分范围为 1（低）到 10（高）。

   a. In some weeks the score reported includes a decimal (e.g. 8.5) because each celebrity performed more than one dance and the scores from each are averaged.

   b. In some weeks, bonus points were awarded (dance offs etc); they are spread evenly across judge/dance scores.

   c. Team dance scores were averaged with scores for each individual team member.

   d. 某些周的得分包含小数（例如 8.5），因为明星当周表演多支舞蹈，所报分数为多支舞的平均值。

   e. 某些周会有加分（例如对决舞等）；加分会均匀分配到各评委/舞蹈评分中。

   f. 团体舞得分会与团队中每位成员的个人得分进行平均。

2. Judges are listed in the order they scored dances; thus "Judge Y"may not be the same judge from week to week, or season to season.

   评委按打分顺序列出，因此"Judge Y"并不一定在不同周或不同季对应同一位评委。

3. The number of celebrities is not the same across the seasons, nor is the number of weeks the show ran.

4. Season 15 was the only season to feature an all-star cast of returning celebrities.

5. There are occasionally weeks when no celebrity was eliminated, and others where more than one was eliminated.

6. N/A values occur in the data set for

a. the 4^{th} judge score if there is not 4^{th} judge for that week (usually there are 3) and

b. in weeks that the show did not run in a season (for example, season 1 lasted 6 weeks so N/A values are recorded for weeks 7 thru 11).

7. A 0 score is recorded for celebrities who are eliminated. For example, in Season 1 the first celebrity eliminated was Trista Sutter at the end of the Week 2 show. She thus has scores of 0 for the rest of the season (week 3 through week 6).

8. 不同季的名人数量不同，节目持续周数也不同。

9. 第 15 季是唯一一季全明星回归赛季。

10. 有时某些周没有淘汰，也有时一周淘汰多于一人。

11. 数据集中出现 N/A 值的情况包括：

    a. 若当周没有第 4^{th} 位评委（通常只有 3 位），则第 4^{th} 位评委评分为 N/A；

    b. 某季中节目未播出的周（例如第 1 季仅持续 6 周，因此第 7 至第 11 周记为 N/A）。

12. 被淘汰选手之后的周次记为 0 分。例如第 1 季中首位被淘汰的是 Trista Sutter，她在第 2 周节目结束时被淘汰，因此第 3 至第 6 周的评分均为 0。

Appendix: Examples of Voting Schemes

附录：投票方案示例

1. COMBINED BY RANK (used in seasons 1, 2, and 28^{a} - 34)

1. 按排名合并（用于第 1、2 季及 28^{a} –34 季）

In seasons 1 and 2 judges and fan votes were combined by rank. For example, in season 1, week 4 there were four remaining contestants. Rachel Hunter was eliminated meaning she received the lowest combined rank. In Table 2 the judges scores and ranks are shown, and we created one possible set of fan votes that would produce the correct result. There are many possible values for fan votes that would also give the same results. You should not use these as actual values as this is just one example. Since Rachel was ranked 2^{nd} by judges, in order to finish with the lowest combined score, she has the lowest fan vote (4^{th} place) for a total rank of 6.

在第 1、2 季，评委与观众投票按"排名"合并。以第 1 季第 4 周为例，当时剩余四名选手。Rachel Hunter 被淘汰，说明她的合并排名最低。表 2 展示了评委评分与排名，我们给出一组可能的观众投票数以产生正确结果。能够得到相同结果的观众投票数还有很多种可能。请勿将这些视为真实值，因为它们仅为示例。由于 Rachel 的评委排名为 2^{nd}，为使其合并得分最低，她需要获得最低的观众投票排名（4^{th}），合计排名为 6。

Table 2: Example of Combining Judge and Fan Votes by Rank (Season 1, Week 4)

表 2：按排名合并评委与观众投票示例（第 1 季第 4 周）

Contestant	Total Judges Score	Judges Score Rank	Fan Vote*	Fan Rank*	Sum of ranks
Rachel Hunter	25	2	1.1 million	4	6
Joey McIntyre	20	4	3.7 million	1	5
John O' Hurley	21	3	3.2 million	2	5
Kelly Monaco	26	1	2 million	3	4

选手	评委总分	评委排名	观众票数 *	观众排名 *	排名之和
Rachel Hunter	25	2	1.1 million	4	6
Joey McIntyre	20	4	3.7 million	1	5
John O'Hurley	21	3	3.2 million	2	5
Kelly Monaco	26	1	2 million	3	4

- Fan vote/rank are unknown, hypothetical values chosen to produce the correct final ranks
- 观众票数/排名未知，这里为产生正确最终排名而设定的假设值

2. COMBINED BY PERCENT (used for season 3 through 27^{a})

2. 按百分比合并（用于第 3 季至 27^{a} 季）

Starting in season 3 scores were combined using percents instead of ranks. An example is shown using week 9 of season 5. In that week, Jennie Garth was eliminated. Again, we artificially created fan votes that produce total percents to correctly lead to that result. The judges'percent is computed by dividing the total judge score for the contestant by the sum of total judge scores for all 4 contestants. Based on the judges'percent, Jennie was 3^{rd} . However, adding the percent of the 10 million artificially created fan votes we assigned to the judges'percent she was 4^{th} .

从第 3 季开始，合并方式改为使用"百分比"而非"排名"。示例取自第 5 季第 9 周，当周 Jennie Garth 被淘汰。同样，我们构造了一组观众投票，使合并百分比正确产生该结果。评委百分比的计算方式为：某选手评委总分除以当周四名选手评委总分之和。按评委百分比，Jennie 排名 3^{rd} 。但将我们构造的 1,000 万观众投票比例加到评委百分比后，她的合并排名变为 4^{th} 。

Table 3: Example of Combining Judge and Fan Votes by Percent (Season 5, Week 9)

表 3：按百分比合并评委与观众投票示例（第 5 季第 9 周）

Contestant	Total Judges Score	Judges Score Percent	Fan Vote*	Fan Percent*	Sum of Percent
Jennie Garth	29	29/117 = 24.8%	1.1 million	1.1/10 = 11%	35.8
Marie Osmond	28	28/117 = 23.9%	3.7 million	3.7/10 = 37%	60.9
Mel B	30	30/117 = 25.6%	3.2 million	3.2/10 = 32%	57.8
Helio Castroneves	30	30/117 = 25.6%	2 million	2/10 = 20%	45.6
Total	117		10 million		

选手	评委总分	评委百分比	观众票数 *	观众百分比 *	百分比之和
Jennie Garth	29	29/117 = 24.8%	1.1 million	1.1/10 = 11%	35.8
Marie Osmond	28	28/117 = 23.9%	3.7 million	3.7/10 = 37%	60.9
Mel B	30	30/117 = 25.6%	3.2 million	3.2/10 = 32%	57.8
Helio Castroneves	30	30/117 = 25.6%	2 million	2/10 = 20%	45.6
Total	117		10 million		

- Fan vote is unknown, values hypothetical to produce the correct final standings
- 观众票数未知，这里为得到正确最终排名而设定的假设值

a The year of the return to the rank based method is not known for certain; season 28 is a reasonable assumption.

a 回归排名法的具体年份并不确定；将其视为第 28 季是合理的假设。